Evolution & Complexity in Business Research

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1 Introduction

This seven-week seminar is about evolution and complexity in business research. The goal of this class is to introduce several techniques (or approaches or technologies or theories) that fit under these relatively new approaches to simulation, research, and exploration that absolutely require today's powerful computers. We will look at the Game of Life, multi-agent models, genetic algorithms, and genetic programming. We will try both to gain a theoretical understanding of each of these techniques and also to get hands-on experience with these technologies. Throughout the seminar we will examine applications of each of these theories to business-related problems. At the end of the seminar students will design an experiment and implement a prototype of the system necessary to carry out the experiment.

2 Basic information

2.1 Time & location

We will meet in the e-Lab (E1434) on Fridays from 9–12noon.

2.2 Books to buy

You should buy these books before the semester begins. You can get them through Amazon or your favorite bookseller.

1. *Turtles, Termites, and Traffic Jams* [62]
2. *Complexity of cooperation* [11]
3. *Growing artificial societies* [19]
4. *Hidden order* [35]

2.3 Other readings

I will pass out the readings for the next seminar each time we meet. You should come by my office sometime on December 18–19 to get the readings for the first class.

3 Schedule

In this section I list readings and Web sites that you can use as resources to learn about that week's technology. I also list several questions that you should be prepared to discuss during *that week's seminar*. (That is, you should complete the readings and think about the questions *before* that week's seminar.) You can always work with someone — and probably should — but you are responsible for learning about everything that is assigned for each week.

3.1 Week 1: Jan 9: Introduction and the Game of Life

This week we're going to get introduced to simulation as a means of experimentation and exploration. We are also going to consider the concept of emergent behavior — what it is and how it comes to be. Finally, we are going to get our hands dirty a little bit by playing with Conway's "Game of Life." This should provide us with insight into both of the above topics and set the stage for more complex techniques and approaches that we will explore later in the semester.

3.1.1 Topics

Introduction to modelling Read the following three articles before continuing with the GoL section. Be sure to refer to the questions in §3.1.2 while you are reading; be prepared to discuss them in class. The first reading is in one of the books that you're buying; the rest you can get from me.

Everyone Everyone should read the following:

1. *Turtles* [62, Chapter 1]
2. Preface to the "Computational Beauty of Nature" [20]

Individual Pick one of the following and read it. You do not have to read the other three articles.

1. Simulation [25, Chapters 1–2]
2. "Why do simulation?" [52]
3. "The world as a process: Simulations in the natural and social sciences" [26]
4. "Simulation and rational practice" [42]

Be prepared to discuss the first two questions in §3.1.2 as they relate to your article for about ten minutes. You may not use slides but you can use a one-page handout if you want. We'll spend some time afterwards trying to come to some sort of consensus about the positions put forth in these articles.

Game of Life I want you to read about and generally play with the following programs and Web sites. The goal is not to become an expert on the GoL but to begin to get familiar with what kind of things can happen when doing a multi-agent simulation. Be sure to define your own initial set-ups and then run them to see what happens. Try to get a feel for the game.

1. "What is the Game of Life?"
   - www.math.com/students/wonders/life/life.html
   - This article provides a very nice introduction to the Game of Life. In addition to describing its intricacies, it also provides both a blank Life grid that allows you to set up your own Game and a set of pre-defined patterns that you can watch evolve.
2. Gardner's *Scientific American* article on "Life" [21]
   - hensel.lifepatterns.net/october1970.html
   - This is the article that introduced the game to a wider audience. This is like a snap shot of another world that existed before the home computer. And it also provides insight into the people who find this sort of thing fascinating (that set would certainly include me).
3. Game of Life exhibit
   - www.collidoscope.com/cgolve
   - This provides some nice examples of just what can be done within the simple rules of Life. Be sure to look in the Pattern Collection under "Rakes and Breeders" and "Ships and Trains" — but don't stop there because there are lots more interesting things to look at.
4. Programs for playing the GoL. At a minimum play with the online tool (as described below). If you have a Windows machine at your disposal, then you should also download and play with Life32 (also described below).
   (a) Online tool for playing with the Game of Life
      - world.std.com/∼bgw/applets/1.02/Life/Life.html
      - A Java applet-based program with which to study the GoL. Some times this page is available and some times it isn't. Play this game at least five times. When you see the game either stop or have only repeating patterns, hit the "Pause" button at

the bottom of the grid, and then write down how many generations have been played and how many are in the population. (This information is at the bottom of the GoL grid.) I got the following:

Generations	Size
287	73
350	77
417	27
370	54
251	59

What would happen if this game board "wrapped around" instead of it having solid borders at the edges?

(b) Game of Life program for Windows (called Life32)

- psoup.math.wisc.edu/Life32.html
- Download this program and the `LifeP.zip` file further down the Web page on which the program is found (under "Get Pattern files"). Open the tutorial by choosing *Help/ Help index* and then clicking on "Tutorial" near the top of the page. Work through the first page or two of the tutorial to figure out what to do. Look at (at least) `breeder` and `max`.
  Note that any time you see a `.RLE` or `.LIF` file you can open it with this application.
- There are some really unbelievable patterns in the subdirectory that contains this application. You might want to begin by looking at `adder`, `ak47`, `aqua20`, `bheptpuf`, `breeder`, and `jaws`. There are many, many others.

(c) GoL program for PalmOS

- www.aldweb.com/articles.php?lng=en&pg=15
- I found this program to be very adequate for playing with the GoL and defining my own patterns. A hint: it helps if you set the cell size to "large" and the speed to "manual."

(d) Linux applications. Do a search for `xlife` or `dblife`. These are distributed as source code.

(e) GoL program for the Macintosh

- trevorrow.com/lifelab/index.html
- I haven't tested this but I assume it works. Let me know if you use it.

5. Variants on the GoL: The GoL has a rigid set of rules; thus, the following are not the GoL but are close relatives of it. The similarity will be obvious. You need not play these as long as you played the original. Play them enough so that you get a feel for how they differ. Be sure to step through

a run (instead of just letting it run as fast as it can) so that you can see exactly what is going on.

(a) Probabalistic Life

- www.abarnett.demon.co.uk/life/index.html
- This is the version that I find to be the most compelling and interesting. It's very much in the spirit of what Conway did, and is probably what he would have done if he could have designed the game assuming everyone had powerful computers at their disposal. (Remember, this was designed as a recreactional mathematics game.)

(b) 3D Game of Life

- www.people.nnov.ru/fractal/Life/Game.htm
- A three-dimensional Game of Life. This has the same types of rules but instead of eight neighbors, these players can have twenty-six neighbors. Be sure to "grab" one of the balls (by clicking down on it) and spin the game around to get different views of it. I find that this version is hard to get any intuition about.

(c) Another 3D Game of Life

- rfhs8012.fh-regensburg.de/∼saj39122/doefe/
- This one is a bit more difficult to play than the above but I found it more interesting overall.

Background for the "Game of Life" You are not required (or even expected) to refer to the following resources; however, you might find them interesting and useful if you are going to continue exploring this topic.

1. GoL objects

(a) Most seen "Game of Life" objects

- wwwhomes.uni-bielefeld.de/achim/freq_top_life.html
- Provides a vocabulary that you can use when talking about recurring patterns that you see in the GoL

(b) Classification of objects

- home.interserv.com/∼mniemiec/lifepage.htm
- Look under the section titled "Object lists...". Thi

(c) Tons of information about objects, etc.

- home.interserv.com/∼mniemiec/lifepage.htm
- Be sure to look under the "Object lists" portion of the page. Each set of items provides a different way of grouping GoL objects. Consider the page "Oscillators and pseudo-oscillators"

(home.interserv.com/∼mniemiec/oscillat.htm). Click on any of the pictures in the page. This will bring up a Web page containing some strange code. This is RLE data. This can be saved to a file and then loaded into a Life program (such as Life32). You can run this game and see what evolves.

(d) GoL status page

- entropymine.com/jason/life/status.html
- Describes what is know about objects in the GoL. Similar information to that shown in the previous resource but organized in a different manner.

(e) Search software

- entropymine.com/jason/life/software/
- This software can be used to search for GoL objects. Not for the faint-of-heart.

2. Turing Machine in the Game of Life

(a) Rendell's Web page describing this specific Turing Machine

- rendell.server.org.uk/gol/tm.htm
- This basically tells us that the rules that define the GoL aren't just interesting — they're sufficiently complex that they can (in theory, and probably in theory only) be used to compute anything that can be computed.

(b) Rendell's original article introducing the Turing Machine

- www.cs.ualberta.ca/∼bulitko/F02/papers/tm_words.pdf
- More detail for those of you whose appetite was merely whetted by the above.

3. Hacker Emblem

- www.catb.org/∼esr/hacker-emblem/
- Eric S. Raymond has proposed that the glider be the emblem representing the entire hacker community.

4. *Wheels, Life...* by Gardner, Chapters 20–22 [22, pp. 214–57]

5. Poundstone's *The Recursive Universe* [61]. A wonderful book about Life and all that can be learned about physics and knowledge by applying its lessons.

Other background Here are some other pages, articles, and books that I'm not expecting you to read but that I've gathered for your further perusal if you're interested.

1. *The structure of scientific revolutions* by Thomas S. Kuhn [45]. If you haven't read this, you should. If not for this class, you should at least read it sometime for your general education.
2. "Dynamic social impact: Robust predictions from simple theory" [47]
3. "Modelling and simulation in the social sciences: From the philosophy of science point of view" [30]
4. "The epistemology of computer simulation" (a class syllabus) www.csse-.monash.edu.au/∼korb/subjects/cse467/467.html.
5. The Big Questions about computer simulations www.csse.monash.edu.au-/∼korb/subjects/cse467/questions.html
6. "The power of simulation" [27]
7. Definition of Occam's Razor [60, 31])
8. "Inductive reasoning and bounded rationality" [5]
9. "Computer simulation of social processes" [24]
10. "Computational models" [36]
11. "The philosophy of science and validation in simulation" [41]

3.1.2 Questions to consider

You should prepare notes (for yourself) for class discussion. I want us to have an informed discussion during this first class. In later classes we will also have formal presentations; however, in this one I simply want to have a discussion

1. Why do we use simulation to study something? Why don't we just use analytical techniques? What does the researcher hope to gain from a simulation?
2. How can we differentiate between good simulation-based research and bad simulation-based research? (BTW, this is an important question and one whose answer you will return to for the rest of the semester.)
3. Do you agree with Resnick's whole "era of decentralization" statement? Is he glossing over anything or ignoring a whole series of changes in the other direction? Any examples of decentralization that he left out or that struck you as being a good example (from your life, classes, or research)?
4. What is the Game of Life?
5. How do the individual cells in a GoL evolve?
6. What causes the complexity of GoLs to appear?

7. Is the GoL deterministic or stochastic? Whatever you answered, what changes to the game rules would change your answer?
8. What is meant by an "emergent phenomena"? Give an example from the GoL.
9. What type of (simplified) social processes might be modelled by the GoL?

I should have asked "What is a simulation?" as the first question.

3.2 Week 2: Jan 16: Multi-agent modelling

This week we're going to look at multi-agent modelling. This approach concentrates on modelling the actions of hundreds of decentralized agents (in the manner of the GoL) that can have relatively complex interactions with each other. The idea is that simple behaviors can result in complex group-level behavior. One of the outcomes of this week is that you'll know how to use NetLogo, a sophisticated general-purpose multi-agent modelling environment.

3.2.1 Questions and tasks

1. Install NetLogo (version 2.0) on your computer.
2. Learn how to use NetLogo by going through the first three tutorials in the NetLogo User Manual (version 2.0).
3. Read the *Turtles* book (149 pages). This book uses StarLogo but I prefer NetLogo. NetLogo is a descendant of StarLogo, implementing several refinements to the StarLogo language. You can use StarLogo if you want but I'll be using NetLogo. See my discussion in §3.2.3. When reading through Chapter 3 in *Turtles*, be sure to have NetLogo and the associated models running so that you can play with them while you read. The important part of reading this book and having NetLogo available to you is that you begin to get a sense of how emergence works. And the only way that I have seen for this learning to occur is to *do* it. Each one of you should take one of the models in Chapter 3 of *Turtles* and be prepared to discuss it in-depth with the class (about 5 minutes each); students taking the class for credit should take two models each. Ensure that all of the models get allocated. Discuss the following in the presentation:
   (a) Did you learn anything interesting about these processes by using these models? This can relate to the computer model techniques themselves or insights you gained from the model. You should assume that the other students have read the book and looked at the model briefly. Your job is to investigate the model in more depth and to facilitate the learning and discussion process.
   (b) Given our discussion from the first class, what reasons do you think there are for creating and running and investigating this simulation? How might the models you investigated be made better science?

I am not looking for a formal presentation; I am looking for evidence that you have been thinking and are able to help others learn.

4. I found the following NetLogo User Community Models to be interesting:

   (a) Artificial Financial Market. This model can be found in the "Community Models" section at ccl.northwestern.edu/netlogo/models/community/Artificial%20Financial%20Market (or the earlier "Herding" ccl.northwestern.edu/netlogo/models/community/Herding)

   (b) Queuing Systems. This actually refers to several different models; they can be found in the "Community Models" section at maths-.paisley.ac.uk/jemg/Queuing%20Models.htm.

   (c) Neteffect (can be found in the "Community Models" section at ccl-.northwestern.edu/netlogo/models/community/Neteffect)

   (d) GameTheory (can be found in the "Community Models" section at ccl.northwestern.edu/netlogo/models/community/GameTheory)

   Each one of you should choose one of the above models (or another one if you find it more interesting or useful) and be prepared to discuss it during the seminar. Argue among yourselves as to who gets to present which model. Students taking the class for credit should choose one of the models listesd above. In your presentation you should explain what it does, what it allows the user to learn about, what lessons you might have learned, and any other interesting observations. Each model discussion should take about 10 minutes. In this case you should not assume that the other students have investigated the model.

5. Did the models you explored or read about provide any inspiration for a model you might explore? As you saw from your readings, this type of model can simulate many different types of processes. Yours might be about the detection of bugs in a program, or the structure of the economy, or — anything at all.

6. What did you learn about how *you* think about centralization and decentralization? Did this book and these models change your thinking in any specific way?

3.2.2 Resources

1. *Turtles* [62].

2. NetLogo and related information

   (a) NetLogo program

   - ccl.northwestern.edu/netlogo/
   - Download and install this software on your computer.

   (b) NetLogo tutorials

- ccl.northwestern.edu/netlogo/whatsnew2_0.html
- This page contains the HTML tutorials, the HTML user manual, and the PDF versions if you want to print them.

(c) NetLogo community models

- ccl.northwestern.edu/netlogo/models/community/
- These models are contributed by the community and aren't official models (whatever that means). On the other hand, some of these models are very interesting and applicable to this seminar.

(d) NetLogo Learning Lab

- netlogo.modelingcomplexity.org
- Resources for the NetLogo modeller located at Mesa State College. You don't need to do anything with this now; I just wanted you to know that it exists.

3. StarLogo

- education.mit.edu/starlogo/
- A Java-based multi-agent modelling system that has a fairly good-sized community supporting it.

3.2.3 Notes if you use NetLogo

NetLogo, while a descendant of StarLogo, differs in many small ways from its ancestor. However, many of these changes are improvements rather than change for change's sake (in my opinion, anyway). Further, and pertinent to this class, the StarLogo models discussed by Resnick in *Turtles* are all available in NetLogo.

The following are a list of the models discussed in *Turtles* and the equivalent NetLogo models (and their location — usually in the NetLogo Models Library):

- **Slime Mold, pp. 50–59**. Biology/ Slime (model can be run at ccl-.northwestern.edu/netlogo/models/Slime)
- **Artificial Ants, pp. 59–68**. Biology/ Ants (model can be run at ccl-.northwestern.edu/netlogo/models/Ants)
- **Traffic Jams, pp. 68–74**. Social Science/ Traffic Basic (model can be run at ccl.northwestern.edu/netlogo/models/TrafficBasic)
- **Termites, pp. 75–81**. Biology/ Termites (model can be run at ccl-.northwestern.edu/netlogo/models/Termites)
- **Turtles and Frogs, pp. 81–88**. Social Science/ Segregation (model can be run at ccl.northwestern.edu/netlogo/models/Segregation)
- **Turtle Ecology, pp. 88–95**. I created this model and uploaded it to the NetLogo Web site (under "Community Models"; ccl.northwestern-.edu/netlogo/models/community/turtle-ecology).

- **New Turtle Geometry, pp. 95–103**. There's no direct translation of this model but there are a lot of related ones in the model library under "Mathematics."

- **Forest Fire, pp. 103–110**. Earth Science/ Fire (model can be run at ccl.northwestern.edu/netlogo/models/Fire)

- **Recursive Trees, pp. 110–116**. I created this model and uploaded it to the NetLogo Web site (under "Community Models"; ccl.northwestern-.edu/netlogo/models/community/recursive-trees).

3.3 Week 3: Jan 23: Social Science from the Bottom Up

This week we're going to look at one researcher's work as it relates to multi-agent modelling. Specifically, Epstein & Axtell's *Growing artificial societies* book will be the centerpiece of this week's work; we will also look at some models based on this work and subsequently derived from it.

3.3.1 Questions and tasks

1. Read all of *Growing Artificial Societies*.

2. Look through both presentations referred to above in resources 5 and 6.

3. Look through the models referred to in resources 2, 3, and 4. You can then work with one more closely if it helps you explore your chapter.

4. You might also find the resources listed in the background section (§3.3.3) helpful when constructing your presentation. I handed out the article listed in resource 2 of §3.3.3.

5. Here's what you'll do next class:

   (a) If you're taking this class for credit, you will be responsible for discussing (for about 25 minutes) one of Chapters 2–7 and its associated model.You can assume that you'll have a computer available to you for both presentation software and showing the model.

   (b) If you're just sitting in this class, you will be responsible for contributing to the discussion of one of the chapters.

   Part of your task when presenting a chapter (and, subsequently, leading the ensuing discussion) is to discuss whether or not this book (and, specifically, your chapter) is good science. If you think it can be improved, present specific recommendations for how it might have been (or could still be) improved. Your presentation should discuss the concepts covered in the chapter and should also demonstrate a model (where appropriate).

3.3.2 Resources

1. *Growing artificial societies* [19]

2. A NetLogo model of SugarScape is at backspaces.net/Models/sugarscape-.html.

3. Another such model can be found at ccl.northwestern.edu/netlogo/models-/WealthDistribution.

4. An updated and expanded version of the previous model can be found on the NetLogo site at ccl.northwestern.edu/netlogo/models/community-/new_wealth_distribution. Here's the description from the NetLogo site: "Expanded version of the Wealth Distribution model in the Sample Models Library. The main changes include the possibility of patches becoming more productive, wealthy turtles now may settle on a patch and hire employees, population may no longer be constant, and turtles can now inherit wealth from their parent." It is also referred to at the Modelling complexity site.

5. A *very* nice PowerPoint presentation related to the SugarScape model can be found at www.modelingcomplexity.org/pols490/sugarscape_p2_03.ppt. It refers to the "new wealth distribution" model you downloaded above.

6. Presentation about this book can be found at www.scs.carleton.ca/∼arp-white/courses/95590Y/notes/SI%20Lecture%2014.pdf.

3.3.3 Background

1. Reviews at Amazon.com. I thought you'd find these interesting.

2. Article in *The Atlantic* about Epstein & Axtell's work wwwṫheatlantic-.com/issues/2002/04/rauch.htm.

3. Artificial Societies, Theory Building, and Memetics can be found at cfpm-.org/∼david/papers/cyber35.doc.

4. PDA Participatory Simulations education.mit.edu/pda/index.htm. These allow people (with the aid of a PDA) to act the part of agents in simulations similar to that done in NetLogo.

3.4 Week 4: Jan 30: Genetic algorithms

This week we're going to look at something (seemingly) completely different. We're going to investigate genetic algorithms, the idea that brought John Holland a MacArthur Fellowship.

3.4.1 Questions and tasks

1. Read "Finding decision rules with genetic algorithms" (here)
2. Read the "Introduction to genetic algorithms" (here).
3. Read *Hidden order*.
4. Be prepared to answer and discuss the following questions:
   (a) What is the purpose of a genetic algorithm?
   (b) What does one particular gene represent?
   (c) How does the population evolve?
   (d) Is the genetic algorithm deterministic or stochastic?
   (e) What purpose does evolution serve?
   (f) Describe the four properties and three mechanisms that are common to all complex adaptive systems and, specifically, how they apply to genetic algorithms.
   (g) How does adaptation come about? That is, how is it that later generations generally produce better-performing individuals than earlier generations?
   (h) What is Holland's Echo model? What does it simulate?
   (i) Think of a problem that a GA might help you solve. Set up the encoding.

3.4.2 Resources

Foundation These resources introduce genetic algorithms.

1. *Hidden order* [35]
2. "Finding decision rules with genetic algorithms," by Jim Oliver, *AI Expert*, March 1994, pages 33-39. HTML version at www.samoore.com/phd2004-/findingDecisionRulesOliver.html; PDF version at www.samoore.com/phd-2004/FindingDecisionRulesOliver.pdf
3. "Introduction to genetic algorithms"
   - cs.felk.cvut.cz/∼xobitko/ga/
   - This article provides the best online introduction to what genetic algorithms are and how they work.
4. Concrete example of a GA
   - www.cs.rochester.edu/users/faculty/leblanc/csc173/genetic-algs/example-.html.

- This short page describes the operations of a genetic algorithm. If you want to know how a population of agents are assigned values, reproduce, mutate, and create a new generation, then this is the place.

5. Java-based GA
   - www4.ncsu.edu/eos/users/d/dhloughl/public/stable.htm
   - This is a Java applet that shows the results of a genetic algorithm. From the page: "This applet is designed to be a tool by which interested people can learn some of the basics of genetic algorithms (GAs) while being able to experiment with a real GA. Users can select values for the GA's search parameters and can modify the parameters while the GA is running to see the real-time effects."
6. GA Tutorial
   - Original version can be found at www.hao.ucar.edu/public/research-/si/pikaia/tutorial/tutorial.ps while a PDF translation can be found at www.samoore.com/teach/phd/2004/gatutorial.pdf
   - This 74-page technical note from the National Center for Atmospheric Research explains genetic algorithms in the context of other optimization techniques. It is a great place to begin if you're interested in doing more in-depth work with GAs.
7. "The hitchhiker's guide to evolutionary computation"
   - surf.de.uu.net/encore/www/
   - This is the FAQ for comp.ai.genetic. It's a little out-of-date with a date of April 2001. It is an organized list of resources (on-line and otherwise) that is a great place to begin your search for basic information related to genetic algorithms.
8. **GA foundations**: [56], [35], [32], [17], [34], [33]
9. Center for the Study of Complex Systems (here at the University of Michigan) www.pscs.umich.edu/
10. Good set of related links www.pscs.umich.edu/education/websites.html

Applications These resources demonstrate applications of genetic algorithms.

1. An adaptive evolutionary approach to option pricing via genetic programming econpapers.hhs.se/paper/fthnystfi/98-086.htm
2. "Using GAs to find technical trading rules" [2]
3. "Direct marketing performance..." [13]
4. "Auctions with artificial adaptive agents" [3]

Software These links contain genetic algorithm software.

1. Source code for GAs from Illinois
   - www-illigal.ge.uiuc.edu/sourcecd.html
   - This is the software page from the Illinois Genetic Algorithm Laboratory (IlliGAL). It has all types of software related to GAs.
2. Source code for GAs from Michigan State
   - garage.cps.msu.edu/software/software-index.html
   - This is software from MSU's Genetic Algorithms Research and Applications Group (GARAGe). This is home-grown software for both GAs and GPs.
3. `C++` source code
   - lancet.mit.edu/ga/
   - This is the home page for *GAlib*. From the Web page: "GAlib contains a set of C++ genetic algorithm objects. The library includes tools for using genetic algorithms to do optimization in any C++ program using any representation and genetic operators. The documentation includes an extensive overview of how to implement a genetic algorithm as well as examples illustrating customizations to the GAlib classes."

Stanislaw Ulam Holland begins the preface to *Hidden Order* with a reference to Stanislaw Ulam. Here are some references to this extraordinary man.

1. Page at the School of Mathematics and Statistics at the University of St. Andrews, Scotland: www-gap.dcs.st-and.ac.uk/∼history/Mathematicians-/Ulam.html
2. Page from Adam Ulam: www.aulam.org/stanulam.htm
3. Describes the Stanislaw Ulam Memorial Lectures: www.santafe.edu/sfi-/publications/ulam.html
4. Amazon information about *Adventures of a Mathematician*.

BACH group Holland mentions the BACH group at the University of Michigan. Here's some information about them.

Arthur Burks Article that discusses his contributions to the creation of the ENIAC: www.the-scientist.com/yr1995/august/birth_950821.html

Robert Axelrod His homepage: www-personal.umich.edu/∼axe/homepage-.html

Michael Cohen His home page: www.si.umich.edu/∼mdc/

John Holland Proceedings of a conference held at UM in his honor: www-.pscs.umich.edu/research/jhhfest-proceedings.html

Carl Simon His home page: www.math.lsa.umich.edu/research/profiles/simon-profile.shtml

Rick Riolo Director of the Computer Lab for the Center for the Study of Complex Systems: www.pscs.umich.edu/PEOPLE/rlr-home.html

3.5 Week 5: Feb 6: Cooperation

The centerpiece of this week's research is Axelrod's *The Complexity of Cooperation*. This wonderful book describes how complexity theory (and, specifically, genetic algorithms) can be applied to game theory.

3.5.1 Questions and tasks

1. Read pages 1–15 of *The Evolution of Cooperation*. This provides some background on the prisoner's dilemma.

2. Read *The Complexity of Cooperation*.

   (a) Everyone needs to read the preface, introduction, and chapter 1.

   (b) Otherwise, please read the following chapters:

Name	Present	Discussants	
Harris	2	3	6
Ning	3	5	A
Jonathan	4	2	6
Sanjeev	5	4	A
Kurt	6	5	7
Erik	7	3	4
Nathan	A	2	7

   You should prepare a 15-20 minute discussion of the chapter in the "Present" column. You should be prepared to contribute to the discussion and react to the presentation of the other two chapters in your row. Use any resources for Axelrod's Web site that might do you good.

3. Read Binmore's review of the book. Be prepared to contribute to a discussion of this review. Specifically, you should focus on preparing comments concerning how you agree and/or disagree with Binmore's arguments and positions.

3.5.2 Resources

1. *The Evolution of Cooperation* [10]
2. *The Complexity of Cooperation* [11]
3. Review of the book by Binmore jasss.soc.surrey.ac.uk/1/1/review1.html.
4. Axelrod's response to Binmore www.samoore.com/phd2004/AxelrodResponse-ToBinmore.pdf
5. Axelrod's page for this book pscs.physics.lsa.umich.edu/Software/Complex-Coop.html.

3.5.3 Related resources

1. "Computer simulations of sustainable cooperation in social dilemmas" [48]
2. "Coevolution of strategies in n-person prisoner's dilemma" [49]
3. "The future of complexity theory for group processes theory and research" http://home.sc.rr.com/bmarkovsky/Papers/ASAComplexity02.pdf.

3.6 Week 6: Feb 13: Economics, finance, & organizations

This week we're going to look at some specific examples of how complexity and evolution have been applied to research problems in economics, finance and organizational theory.

3.6.1 Tasks

We're getting to the end of the semester and are done learning about new research tools. We're also getting closer to that point in time when you need to begin thinking about the paper you're going to write for this class.

1. As a step in this direction, this week you should pick one article that interests you — and that is, perhaps, related to the paper that you will be writing — from the materials that I bring to class.
2. Let me know which article you are picking. I will get copies made of this article.
3. Create a presentation in which you summarize the article for us. Highlight its research questions, its research method, its findings, and any open research questions that might be of interest to you or your audience. This should be about 15 minutes. You should not assume that anyone else has read the article — because they probably will not have read it. Don't think that you're trying to convey all the details of the paper to us in this time; you should think about selling the paper to your audience, describing what is most interesting about it and why we would want to read it.

4. Begin to think about the paper that you're going to write for this class. Write up a half-page of your ideas about what this paper might be. We'll spend a few minutes in class discussing each one of your ideas and seeing if there are ways in which your proposal can be improved or clarified.

3.6.2 Resources

Main Here are the books that you should look through when looking for an article. I have them all in my personal library.

1. *Barriers and bounds to rationality* by Albin and Foley
2. *Dynamics of organizations* by Lomi and Larsen
3. *Simulating social phenomena* by Conte, Hegselmann, and Terna
4. *The Economy as an Evolving Complex System* by Anderson, Arrow, and Pines
5. *The Economy as an Evolving Complex System II* by Arthur, Durlauf, and Lane
6. *Evolutionary computation in economics and finance* by Chen

Original These are the original resources that I had listed in this section. I didn't want to lose this list so here they are.

1. "Asset pricing under endogenous expectations in an artificial stock market" [8]
2. "Natural Rationality" [16]
3. "How economists can get alife" [65]
4. "Decentralized exchange" [1]
5. A hands-on approach to evolutionary simulation: Nelson and Winter models in the Laboratory for Simulation Development econpapers.hhs.se-/article/jemejemed/1003.htm
6. Managing information complexity in a supply chain model by agent-based genetic programming econpapers.hhs.se/paper/scescecf1/238.htm
7. "Cooperative computation" [38]
8. "Option pricing" [15]
9. Links to evolutionary economics can be found at www.business.auc.dk-/evolution/evolecon/evolinks.html?tchoice=computational&choice=all
10. Page of readings and links for agent-based computational economics www-.econ.iastate.edu/tesfatsi/asocnorm.htm.

11. *The edge of organizations* [51]

12. *Dynamics of organizations* [50]: Chapters 7 & 10 are particularly interesting (my opinion, anyway).

13. *Computational modeling of behavior in organizations* [37]

14. *The economy as an evolving complex system (II)* [7]

15. *Development, geography, and economic theory* by Paul Krugman [44]. I think that this book describes an area of economic research that could be addressed very well by multi-agent systems. It provides a nice description of a problem that could be developed into publishable papers (if it hasn't already for other researchers).

3.7 Week 7: Feb 20: Complexity

This last week of the seminar we're going to attempt to summarize our understanding of complexity and evolutionary processes and how they relate to business research.

3.7.1 Questions and tasks

1. Bring a 1-2 page outline or proposal to class. It should be a more complete fleshing out than what you gave me for the previous class. It should provide some pointers to the body of theory that you will be referring to. It should also describe the general type of model you'll be looking at as well as the questions you hope to investigate.

2. Read the Page paper. Be prepared to lead a discussion or be a primary discussant of the letters below. (Figure out what I mean!) In leading a five minute discussion of this letter, reflect back on the semester. Think of other models or readings or techniques or whatever that seem especially appropriate. What have we learned about this topic?

Name	Lead	Discussant
Kurt	ahov	bjrz
Ning	bipw	cksq
Sanjeev	cjqx	dlmx
Erik	dkry	etny
Jonathan	elsz	aiqy
Harris	fmt	ugov
Nathan	gnu	fhpw

3. Read the Epstein paper. Reflect on the following questions and be prepared to participate in a discussion:

- How can agent-based computational models be used in business research? What general types of problems can it help with? What types of questions can it help answer. Use Epstein's paper to help you out.
- Come up with two or three (or more) topics that you think would benefit from this type of research.
- What does this approach add that standard approaches can't? Again, use the many clues provided in Epstein's paper to help you out with this question.

4. Read the Arthur paper. Why? Just because I think you'll get something out of it.

3.7.2 Resources

Main These are the articles that you need to read for next class.

1. "Why do things become more complex?" by W. Brian Arthur [4], www.samoore.com/phd2004/whycomplex.pdf
2. "Computational models from A to Z," by Scott E. Page, www.santafe.edu/projects/swarm/swarmfest99/keynote.html (PDF version at www.samoore.com/phd2004/compmodelspage.pdf). For UM students, the published version in *Complexity* can be downloaded at www3.interscience.wiley.com/cgi-bin/fulltext/66003738/PDFSTART.
3. "Agent-based computational models and generative social science" by Joshua Epstein, www.samoore.com/phd2004/generativess.pdf

Other These are other articles that I've found useful.

1. "Landscape theory" [9] (also as [11, Chapter 4])
2. "What is complexity?" [23]
3. Review last chapter of *Turtles*
4. A review of *Turtles* [66]
5. "Self-organized criticality" [12]

4 Paper

Except for your preparations for each class, your only assignment for this seminar is for you to write a paper. This paper should do the following:

- Outline a set of related research questions that you think would benefit from a simulation model.

- Describe what existing theory, experiments, and/or empirical research have told us about the answers to these questions. Provide references to underlying theory that show the need for your model.
- Describe a model (multi-agent, GA, etc.) that could be used to address the above question(s). The more details that you can provide here, the better off you'll be.
- Is there an underlying theory that this model is referencing or embodying? Be sure to describe this relationship (if there is one). If there isn't one, then be prepared to defend any charges that this model is just *ad hoc* and doesn't really tell us anything.
- Describe why you think the model helps you answer the questions you're interested in.
- Implement a basic version of this model. This isn't a requirement to pass the class but it's recommended.
- Describe any *preliminary* findings that you have come across from this model.

At the end of the semester (e.g., in April) you will turn in to me the paper and the model. At any time in the meantime, you can come by and talk with me about your progress. I am also very interested in working with any of you in turning these papers into publications. When you turn in the paper, let me know if this is something that you're interested in.

5 Resources for class

5.1 Related readings

General [25], [20]

Evolutionary economics [58], [55], [6]

Complex systems [19], [10], [11], [63]

Software

- **Complex systems**: NetLogo [67], RePast [64]

GA applications [14], [2], [13], [40], [3]

GP foundations
1. [43]
2. www.genetic-programming.org/
3. www.geneticprogramming.com/Tutorial/index.html

GP applications [14], [57], [54], [18]

Other [39], [53], [46]

A Cellular automata

The following are links that provide some insight into cellular automata (CA). Since the GoL is a variant of CA, I decided to leave this topic out of our seminar.

A.1 Resources

1. Introduction to cellular automata
   (a) One-dimensional cellular automata
      - godel.hws.edu/xJava/CA/CA.html
      - asfd
   (b) More formal, academic treatment of CAs
      - www.ifs.tuwien.ac.at/∼aschatt/info/ca/ca.html
      - asdf
   (c) asdf
      - www.rennard.org/alife/english/acintrogb01.html
      - asdf
   (d) asfd
      - cell-auto.com/definition/index.html
      - asdf
   (e) asfd
      - www.cs.bgu.ac.il/∼sipper/ca.html
      - asdf
   (f) asdf
      - homepages.feis.herts.ac.uk/∼nehaniv/CM/ca.html
      - asfd
2. "Understanding complex social dynamics" [29]. An article that argues for using CAs in social science research.
3. "Modeling social change with cellular automata" [59]
4. "Cellular automata in the social sciences: Perspectives, restrictions, and artefacts" [28]
5. asdf
   - www.collidoscope.com/modernca/
   - Really, really great online CA software.
6. Life-like rules and objects
   - entropymine.com/jason/life/alt/

- You can download some of the patterns (the `.lif` files) into Life32 and see what they do.

7. Langton's Ant: users.libero.it/acnard/ant.html
8. CA and the edge of chaos: math.hws.edu/xJava/CA/
9. "Exploring emergence," by Mitchel Resnick and Brian Silverman
   - llk.media.mit.edu/projects/emergence/index.html
   - A nice overview of CAs and the idea of emergence.
10. CA software
    (a) Java applet
        - www.mirekw.com/ca/mjcell/mjcell.html
        - MJCell
    (b) Java applet and application
        - jmge.net/java/csprings/
        - Cellsprings
    (c) Windows software
        - www.fourmilab.ch/cellab/
        - CelLab.
    (d) X windows
        - quark.phy.bnl.gov/www/xtoys/xtoys.html
        - asdf
    (e) Python-based CA simulation package
        - www.alcyone.com/pyos/cage/
        - asdf
    (f) CA generator (C++ source code)
        - kidojo.com/∼yebb/cellauto/
        - asdf
    (g) NetLogo and CAs
        - www.itee.uq.edu.au/∼comp4001/Tutorials/tut2-netlogo/netlogo-tutorial.html
        - asdf
11. A simulation for investigating flow of power in a power grid
    - www.cs.sjsu.edu/faculty/rucker/capow/
12. Ed Fredkin's Digital Philosophy
    - digitalphilosophy.org/

- asdf

13. Early references for CA
    - members.aol.com/life1ine/life/bib.htm
    - If you become interested in CAs, then this list provides a good place to start building your CA library.
14. Digital, Amorphous Physical Models
    - swiss.csail.mit.edu/∼rauch/dapm/
    - A physical model similar to CA but it doesn't require that there be a grid or synchronized actions.
15. Complete set of CA links: cell-auto.com/links/

A.2 Questions

1. What distinguishes a CA from the GoL?
2. How do the individual cells in a CA evolve?
3. What causes the complexity of CAs to appear?

B Other possible topics

1. *Computational Beauty of Nature* by Gary William Flake mitpress.mit.edu-/books/FLAOH/cbnhtml/other.html
2. SWARM (UM software)
3. Swarm (technique)
4. ALife
5. Scott's work
6. Koza introduction.
7. Evolving cellular automata: The home page for this research project is at (www.santafe.edu/projects/evca/); a list of papers can be found at www.santafe.edu/projects/evca/Papers/papers.html; a good paper titled "Evolutionary unfolding of complexity" can be found at (www.santafe.edu-/projects/evca/Papers/teuoc.html)

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